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MULTIFRACTAL ANALYSIS AT NEGATIVE q

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13. ABSTRACT *(Maximum 200 words)*

A box-based correlation integral algorithm for the determination of generalized fractal dimensions is described. The algorithm has been tested for Euclidean curves, Koch symmetric and asymmetric triadic snowflakes, split snowflake halls, and the multifractal construction based on Mandelbrot's 13-element generator. The technique is well-suited to the analysis of experimental data with inherent uncertainty, it is efficient, and it converges to analytic results for the model point sets at all q values.

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INTRODUCTION

Block, von Bloh, and Schellnhuber (BBS) (ref 1) presented a box-counting algorithm for the measurement of generalized (fractal) dimensions, which they refer to as "Efficient Box-Counting" (EBC). Meisel, Johnson, and Cote (ref 2) described a box-counting algorithm, which is particularly suited to the analysis of experimental data with inherent uncertainty and large data sets, which they referred to as "Agglomeration Box-Counting" (ABC). Atmanspacher, Scheingraber, and Wiedenmann (ref 3) applied techniques similar in spirit to EBC and ABC, which they referred to as "histogram techniques," and a technique based on the generalized correlation integral (ref 4), to the determination of the multifractal properties of the distribution of galaxies on the celestial sphere (ref 3).

It was shown in Reference 2 that although box-counting algorithms converge to analytic results for model fractal point sets at $q \geq 0$, they fail to converge to analytic results at $q < 0$. Similarly, Atmanspacher et al. (ref 3) reported consistent results for the left branch of the $f(\alpha)$ spectrum ($q \geq 0$), but noted discrepancies for the right-hand branch ($q < 0$). It is evident that a reliable algorithm for the determination of $D(q)$ at $q < 0$ or equivalently for the determination of the right-hand branch of the $f(\alpha)$ spectrum is urgently required.

The present work describes a Box-Based Correlation Integral (BBCI) method for multifractal analysis. The BBCI algorithm has been tested for a selection of model multifractal point sets, which are subject to simple analysis along the lines in Halsey et al. (ref 5). BBCI values of $D(q)$ converge close to analytic values at all $q \in [-25,25]$ for the model multifractal point sets. The algorithm is well-suited to experimental data application, and it converges at least as rapidly as ABC at $q > 1$.

THEORY

The Box-Based Correlation Integral (BBCI) algorithm is implemented as follows:

1. Define a set of nested hypercubes ("boxes") appropriate for the given (or anticipated) point set S. We refer to these boxes as elementary hypercubes or elementary boxes. For experimental data, the appropriate choice of elementary box edge E_0 should reflect the inherent uncertainty of the coordinates of the point set within limits set by storage requirements.

Storage requirements are effectively determined by the choice of elementary hypercubes rather than the size of the point set. Thus, large point sets can be accommodated with relatively modest resources.

2. Compute or measure the occupation numbers n_i for each of the elementary hypercubes. Define N as

$$N = \sum_{i}^{elementary\ boxes} n_i .$$

In our tests N ranged between about 10^3 and 10^9.

3. Define a reference set that comprises a subset of the <u>occupied</u> elementary boxes. Let N_{ref} be the number of elementary boxes in the reference set. In our tests with $N = 768^2$ elementary boxes, we took n_{ref} as the smaller of the total number of occupied boxes or 15,000.

4. For each member of the reference set, define a set of hypercube edge lengths, $E = (2n+1)\,E_0$, where $n=0,1,2,...n_{MAX}$. In our tests with $N = 768^2$ elementary boxes, we took $n_{MAX} = 24$.

5. For each q of interest:

a. Compute the box-based generalized correlation integrals $C(q,E,E_0)$, which are defined as

$$C(q,E,E_0) = \{\frac{1}{N_2}\sum_{r}^{N_{ref}} [\frac{1}{N}(\sum_{j}^{elementary\ boxes} n_r n_j G(E,E_0,\mathbf{x}_r-\mathbf{x}_j) -1)]^{q-1}\}^{\frac{1}{q-1}}$$

for the E values defined in step 4, where r runs over the reference set, j runs over all elementary boxes,

$$N_2 = \sum_{r}^{N_{ref}} n_r$$

and the function

$$G(E,E_0,\mathbf{x}) = 1,\ if\ |x_i| \le (E-E_0)/2,\ for\ all\ components\ of\ \mathbf{x}$$
$$= 0,\ otherwise,$$

selects the elementary boxes contained in larger hypercubes of edge E. The vectors x_r and x_j point to elementary hypercubes rather than members of the point set. Reducing the sum over elementary boxes in the side E hypercube by unity eliminates self-correlation contributions.

The box-based generalized correlation integral reduces to the standard generalized correlation integral (ref 4) in the case when the coordinates of the members of the multifractal subset in question are precisely known, the elementary hypercubes are small enough to contain at most one element of the multifractal subset, and the covering set is comprised of hypercubes. That is,

$$C(q,E) = \lim_{E_0 \to 0+} C(q,E,E_0)\ .$$

Computation was slowed down and convergence was not improved by employing (approximate) hyperspheres rather than hypercubes. Also note that special techniques are required near q = 1; here, we restricted q such that $|q - 1| > ½$.

b. Obtain D(q) and the corresponding rms error in D(q) by linear regression on $\underline{\ln}[C(q,E,E_0)]$ = const + D(q) $\underline{\ln}[E]$ for $E \varepsilon \{(2n+1) E_0 | n=1,2,...n_{MAX}\}$.

RESULTS AND CONCLUSIONS

The BBCI algorithm has been tested for Euclidean curves, Koch symmetric and asymmetric triadic snowflakes, split snowflake halls, and the multifractal construction based on Mandelbrot's 13-element generator. The D(q) determined by BBCI converged to values near the analytic D(q) values for q ranging between -25 and +25 in all cases as N was increased.

Figure 1 shows results of application of the BBCI algorithm to split snowflake halls at five values of q. The algorithm was applied to a randomly-oriented construction centered on 768 by 768 square boxes; N_{ref} was the lesser of 15,000 or the total number of occupied boxes. The horizontal lines are the analytic values. Convergence was within about 2 percent for $q < 1$ and within 1 percent for $q > 1$ in this test. Note that convergence is about an order of magnitude faster for $q > 1$ than it is for $q < 1$. Except that convergence is faster for constructions with lower D(0), Figure 1 is typical of convergence obtained by application of the BBCI algorithm to the constructions tested.

Figure 2 shows the results of application of the BBCI algorithm as in Figure 1 for $N = 2.14\ 10^8$ and analytic values of D(q) versus q for split snowflake halls. The increase in the error in D(q) with $|q|$ and the order of the discrepancies are apparent.

The rms errors found in the linear regressions were of the order of the differences between analytic and BBCI results for D(q). An expanded discussion of the results of application of BBCI to model fractals and of the effects of variations of N_{ref}, number of elementary boxes, and n_{MAX} will be presented in a future publication.

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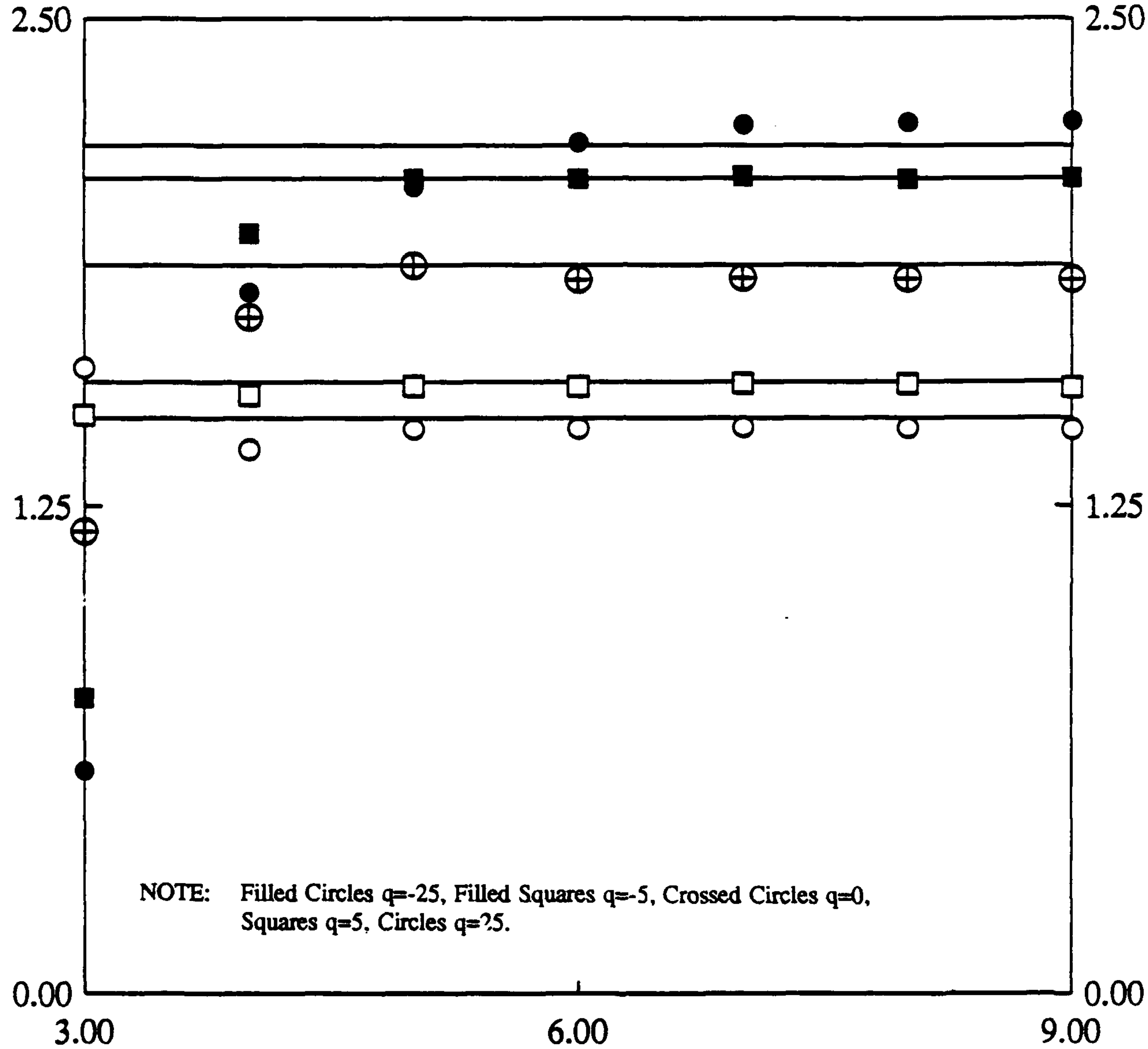

Figure 1. Analytic and BBCI measurements of the fractal dimension D(q) versus level for randomly-oriented split snowflake halls. (The number of points in the construction $N = 11^{level}$.)

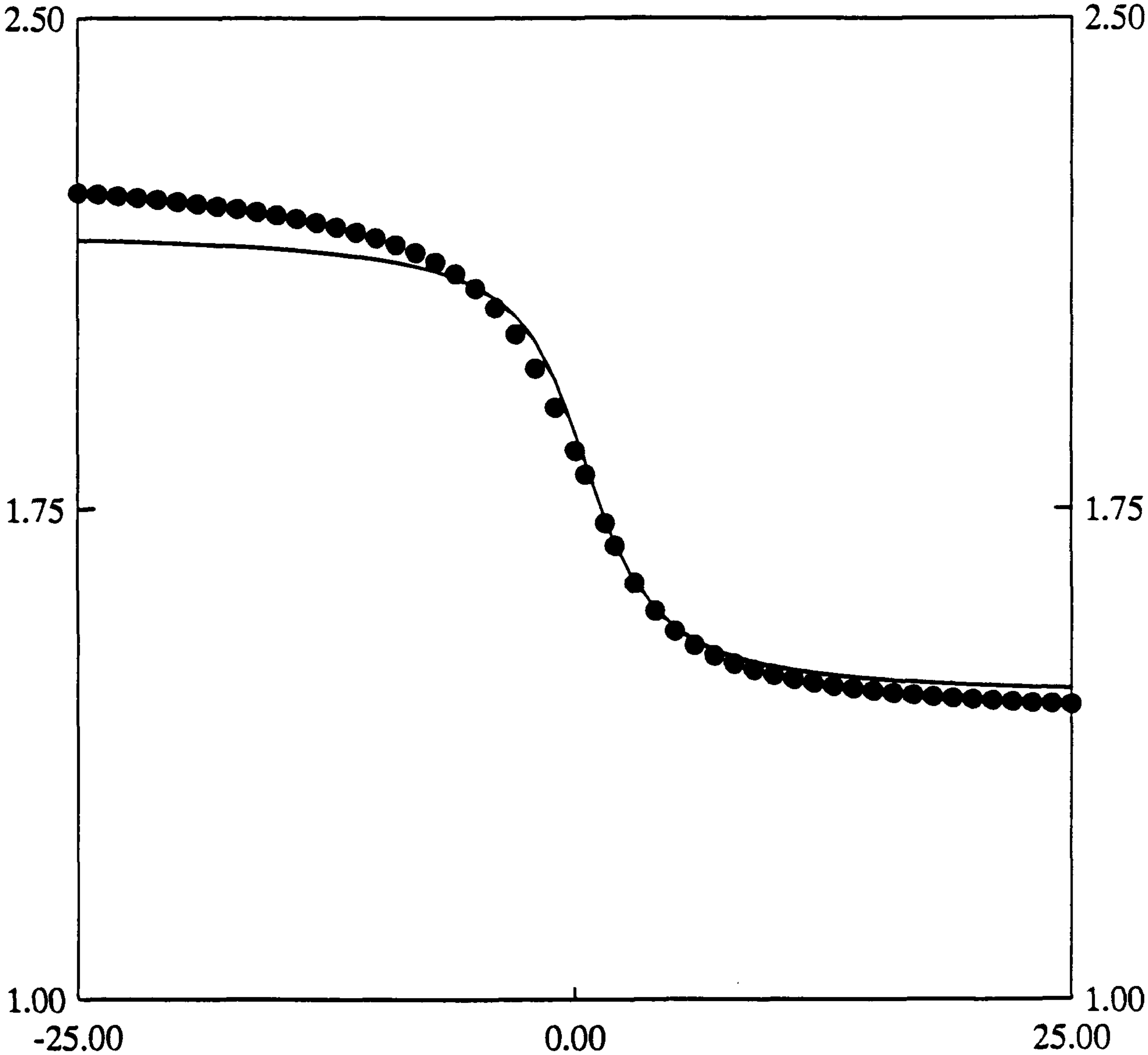

Figure 2. Analytic and BBCI values of the fractal dimension D(q) versus q for level 8 split snowflake halls. The line is the analytic curve.

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